HOW TO GRAPH POLAR FUNCTIONS OF THE FORM $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$
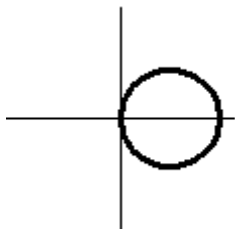
1. Determine the shape of the graph by finding $\left|\frac{a}{b}\right|$.

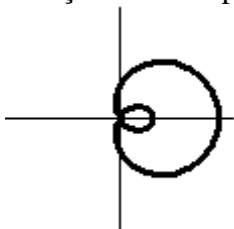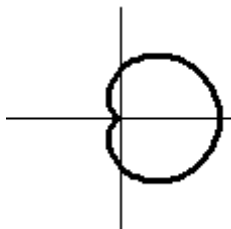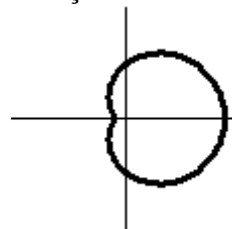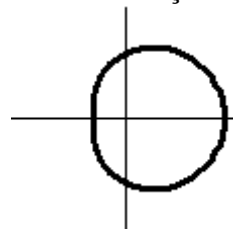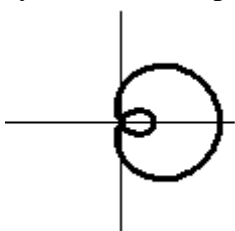| $\left|\frac{a}{b}\right| = 0$	$\left|\frac{a}{b}\right| < 1$	$\left|\frac{a}{b}\right| = 1$	$1 < \left|\frac{a}{b}\right| < 2$	$\left|\frac{a}{b}\right| \geq 2$
circle	limaçon with loop	cardioid	limaçon with dimple	convex limaçon

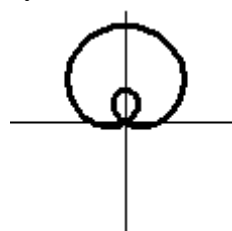2. Determine the axis of symmetry by the trigonometric function used.

$r = a \pm b\cos\theta$
symmetric over polar axis (x – axis)

$r = a \pm b\sin\theta$
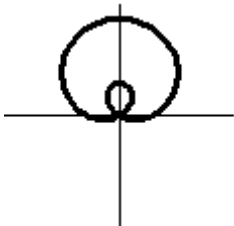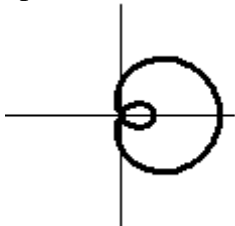symmetric over $\theta = \frac{\pi}{2}$ (y – axis)

3. Determine the "direction" of the graph by the sign of b.
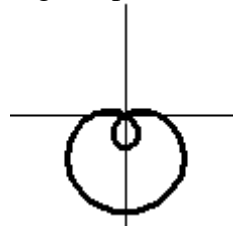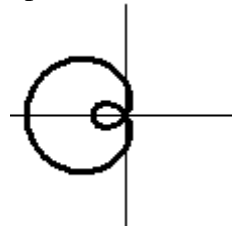
$b > 0$
"bigger" end on the right/top
"puckered" end on the left/bottom

$b < 0$
"bigger" end on the left/bottom
"puckered" end on the right/top

4. Determine the x – and y – intercepts by finding the points corresponding to $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$\theta = 0, \pi$ correspond to positive and negative x – intercepts (assuming $r > 0$)
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ correspond to positive and negative y – intercepts (assuming $r > 0$)

If $r < 0$, the intercept is on the "other side" (negative vs positive, and vice versa) of the corresponding axis

Example: Graph $r = 2 - 3\sin\theta$
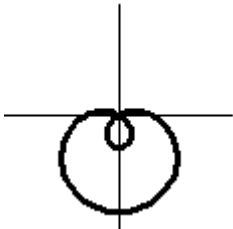
1. $\left|\frac{2}{-3}\right| < 1$ limaçon with loop

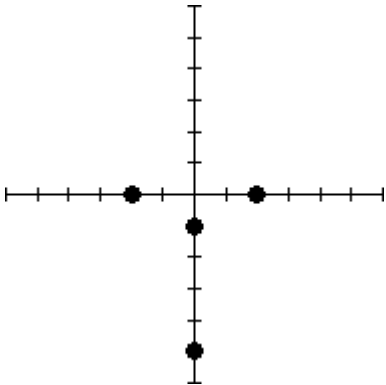2.   equation uses $\sin\theta$ symmetric over $\theta = \frac{\pi}{2}$ (y – axis)

3. $-3 < 0$ "bigger" end on the bottom, "puckered" end on the top

4.

θ	$r = 2 - 3\sin\theta$
0	2 (positive x – intercept)
$\dfrac{\pi}{2}$	-1 (negative y – intercept)
π	2 (negative x – intercept)
$\dfrac{3\pi}{2}$	5 (negative y – intercept)

Intercepts only   Complete graph